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Can Students Predict When Imagery Will Allow Them to Discover the Problem Solution?

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Three studies have been carried out to identify to what extent and on what grounds undergraduates realise that visual imagery can be useful to solve a problem. A series of problems had been selected according to the kind of task (logical, mathematical, geometrical, and practical) and to the objective level of imagery efficacy (as reported by previous experiments aimed at assessing the effects of mental visualisation on problem solving). Then, by means of different procedures, students were asked to rate how useful, in their opinion, mental visualisation is in solving each of such problems and to explain why they thought it is useful. Results showed that usefulness scores were highly consistent and were not affected by gender, course of study attended, and individual differences in imagery (vividness, control, and cognitive style). Undergraduates rated imagery as useful above all when problems deal with concrete situations, whereas mental images were not perceived as possible aids for abstract or conceptual problems. The subjective evaluation did not correspond to the objective effect. However, when a specific imagery strategy was described, the discrepancy between the subjective and objective measures decreased.

Several findings suggest that transformation and synthesis of mental images can lead to the discovery of emerging meanings that induce insightful ideas or help create new products (Helstrup & Anderson, 1996; Roskos-Ewoldsen, Intons-Peterson, & Anderson, 1993). Why does imagery favour mental discovery? It has been argued that the visual representation of a situation allows the flexible manipulation of the elements in unusual ways and that it is a relevant strategy to restructure the material given. It was also proposed that the spatial character of visual images makes them directly accessible to intuitive abilities. Further-

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more, visualisation helps people to have an overall picture of the situation and to catch the structural schema underlying it. Finally, it has been maintained that imagery facilitates the parallel, simultaneous processing of information, which may result in the reduction of cognitive strain, especially under conditions of high mental load (Helstrup, 1988; Kaufmann, 1985, 1988).

These claims have been supported by experimental evidence (e.g. Adeyemo, 1990, 1994; Antonietti, 1991; Barolo, Masini, & Antonietti, 1990; Houtz & Frankel, 1992; Kaufmann, 1990; Martinsen & Kaufmann, 1991), which, however, showed that the heuristic role of imagery in problem solving is restricted to particular situations (Denis, 1991; Richardson, 1983). Restrictions concern, on the one hand, the phase of the solution process in which imagery occurs. In fact, positive effects of visualisation on problem solving are higher when individuals are instructed to generate mental images after being given the problem, rather than when they receive such a hint before the problem is presented (Antonietti, Cerana, & Scafidi, 1994). This suggests that imagery generation is particularly useful when it is orientated toward a specific endpoint, namely, when visual images are created and manipulated by having in mind the goal to be achieved; images produced without the constraints and the directions imposed by such a goal can be interpreted and transformed in a wide range of ways, so that individuals are likely to follow unproductive lines of thinking or fruitful lines of thinking are disrupted.

On the other hand, imagery efficacy in problem solving seems to depend on both objective and phenomenological features of the stimuli, e.g. the degree of complexity of the task, the nature (ill-defined vs well-defined, ambiguous vs non-ambiguous, and so forth) of the problem, and its difficulty. As far as this issue is concerned, Kaufmann and Helstrup (1985) introduced the notion of "level of programming" as a major determinant of the functional usefulness of imagery in problem solving. A problem is programmed if people possess one or more procedures to handle it and, consequently, they do not need further information to proceed; a problem is non-programmed when it appears unstructured and/or requires that new information is acquired. When, in a non-programmed task, abstract or verbal processing leads to a block, imagery—by operating as a support system—may yield a rearrangement of the problem elements so that reasoning can go on productively. Visualisation may prove useful also in situations demanding novel or different approaches rather than routinised strategies; in these cases imagery induces persons to break away from well-established, but not relevant, procedures.

In further developments of this perspective, Helstrup and Kaufmann (1999) claimed that the relative importance of imagery in the solution process is high with unfamiliar problems and decreases with increasing problem familiarity. When a novel problem is given, individuals can use visualisation to transform the situation in a perceptual-like manner so that new information can be identified and rough-outline drafts of potential solutions are designed. However, in some investigations hints at mentally visualising the scene of unfamiliar problems either had no effects or inhibited performance (e.g. Antonietti et al., 1994). Thus, it seems that imagery plays a facilitating role in particular sets of novel problems. For example, performance is improved when the task induces automatic application of procedures, which lead people to ignore some crucial elements, consequences, or alternative representations of the problem: In such cases visualisation enables people to overcome the tendency to make use of complex, unnecessary procedures, such as calculations, theorems, and rules. Mental visualisation helps reasoning also when the main obstacle is fixedness, that is, the tendency to consider some critical elements of the problem in their common role or when it is difficult to consider simultaneously many aspects of the problem (Antonietti, 1991).

The fact that visual images are useful only in some kinds of task stresses the need that, in order to employ mental images in problem solving productively, people should be able to identify such tasks; otherwise imagery either remains an "inert" resource (if it is not exploited in situations for which it is adequate) or is detrimental (if a person relies on it when it is not relevant). For instance, given the problem "A passenger fell asleep after he had travelled half way to his destination. When he awoke there remained half the distance he had travelled while asleep. For what part of the way did he sleep?", someone might be helped to reach the solution by visualising a line or a strip (corresponding to the duration of the entire journey) and partitioning it into parts (corresponding to the time spent, respectively, asleep and awake), so that the ratio between the sleeping and being awake periods is clear (Frandsen & Holder, 1969). However, if such a person believes that, since the problem deals with numbers, only mathematical procedures or abstract concepts should be relevant, he or she should fail to take advantage of possible intuitions suggested by visualisation.

Do individuals have metacognitive competences that allow them to discern in which situations mental visualisation can facilitate problem solving? This metacognitive awareness seems to require adequate opinions about the role of visual images in thinking. However, as yet we have no information about people's beliefs concerning the utility of imagery in solving problems. The studies reported in the present paper were aimed at investigating to what extent and on which grounds people realise that visual imagery can be useful to solve a problem.

STUDY 1

The aims of the study were:

- to understand in which kinds of problems people consider mental visualisation useful to reach the solution
- to identify in which areas people think that visual imagery can help problem solving
- to assess whether conceptions about the usefulness of visualisation are influenced by gender and by course of studies attended by participants
- to verify whether the subjective evaluation of imagery utility matches the actual power of imagery shown by literature.

Method

Participants

Ninety undergraduates (thirty-one males and fifty-nine females) in different disciplines volunteered for the study. They ranged in age between 19 to 24 years. Students were divided into three groups according to their course of study: Economics, Humanities, Social Sciences. Participants were also classified according to the year of course attended: first, second–third, and fouth. They were neither paid nor received course credits.

Materials and Procedure

Literature about the relationships between imagery and problem solving was reviewed[1] and all experimental studies in which an imagery condition was compared to a non-imagery condition were selected. Studies in which imagery was combined with other manipulations (for instance, mood or incubation) were excluded, as well as experiments involving special sets of the population (elderly people, individuals affected by mental retardation, learning disabilities, and so on). Only studies employing pen-and-pencil or verbal problems were considered; moreover, studies that failed to report completely the text of the problems used or whose text was too long to be transcribed onto a single page were left out. Problems that fitted these criteria were divided into

[1]This review was based on the PsycLIT database (1887–1996) through a search in which the keywords "imag*", "visual*", and "problem sol*" were employed and on further papers not included in such a database but quoted in the reference list reported on papers found in the database. Papers written in languages unknown to the author (Russian, Japanese, and so on) were omitted. Finally, experiments concerning three-term-series problems were excluded because they constitute an autonomous field of research where the role of imagery is controversial.

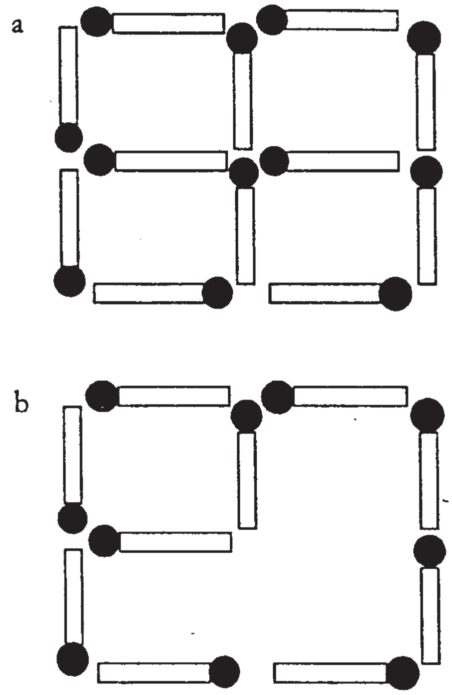

FIG. 1. The two-square problem (a), and solution (b).

four categories according to the type of problems investigated: geometrical, mathematical, practical, and logical. The final restricting criterion was the quotation, in the text or in the tables of the study, of measures of solution rate both in the imagery and non-imagery conditions.[2] Three problems for each category were chosen. An attempt was made to include in each category problems in which visualisation plays a putative facilitating role for different reasons: helping parallel processing, highlighting structural relationships, prompting unusual transformations, and so on. The selected problems were the following.

Geometrical Problems

Square-rhombus problem. "What is the ratio between the area of a square and the area of the rhombus inscribed in the square?" (Kosslyn, 1983). A picture of the figures described in the instruction was given.

[2] Unfortunately, no study involving logical problems met this criterion. Because this category seemed to be relevant to the investigation of the subjective evaluation of imagery usefulness, it was decided to include also logical problems in the studies to be carried out.

Two-square problem. "Look at this figure (Fig. 1a). Obtain two squares by subtracting two matches" (Mosconi & D'Urso, 1974). The solution is described in Fig. 1b.

Triangle problem. "The length of the diameter of a circle is given. How long is the hypotenuse of the triangle inscribed in the upper left quadrant of the circle?" (Köhler, 1969). Also in this case a picture was given.

Mathematical Problems

Sock problem. "In a drawer you have 5 white socks and 5 black socks. What is the smallest number of single socks which you would have to take out to be sure of getting a matching pair?" (Mosconi & D'Urso, 1974).

High-tide problem. "A rope ladder was hanging from a boat so that the laddder had six rungs above the sea. The distance between any two rungs was 30cm. At high tide, the sea level rose 70cm. How many rungs were above the sea at high tide?" (Mosconi & D'Urso, 1974).

Alarm-clock problem. "John went to sleep at 8pm, having previously wound up his old alarm-clock and set the hand to wake up at 9am. He slept soundly until the alarm rang. How many hours' sleep did John get?" (Raudsepp, 1980).

Practical Problems

Candle problem. "Suppose you have to hang two candles on the wall to light them. You have at your disposal some elastic, a box of nails, a hammer, and some matches" (Duncker, 1963).

Rope problem. "Suppose you are in a room where two cords are hung from the ceiling. The two cords are of such a length that when you hold one cord in either hand, you cannot reach the other. Your task is to tie the ends of these cords together. The room is empty. You have only a bunch of keys" (Maier, 1930).

Pea problem. "Two bowls are set on a table, one within the subject's reach and one further away. One bowl contains a number of peas and the other is empty. Also on the table are a newspaper, scissors, paper clips, and rubber bands. How can you, using the material provided, transfer the peas from the filled to the empty bowl without leaving the seat?" (Raaheim, 1974).

Logical problems

Syllogistic problem. The following two premises were given: "All poachers are hunters" and "Some hunters are not vegetarian". Subject had to identify the correct inference within a set of five possible responses.

Cryptoarithmetic problem. "Substitute numbers for letters in the expression DONALD+ GERALD= ROBERT assuming that D= 5" (Bartlett, 1958).

Missionary problem. "Transport three missionaries and three cannibals from one side of river to the other side by using a boat which can carry at most two people and by avoiding a situation that cannibals outnumber the missionaries" (Thomas, 1974).

These 12 problems were presented in a booklet in a randomly assigned order. Each problem was presented on a separate page. The series of problems was preceded by a one-page description of what mental visualisation is, its occurrences in thinking, and its possible role in problem solving. Such a description was as follows:

> Imagery is a kind of mental representation which can represent objects, persons, scenes, situations, words, discourses, concepts, argumentations, and so on in a visuo-spatial format. Mental images can refer to entities that a person: (a) is perceiving at present, (b) has perceived previously, or (c) has never perceived. Mental images can represent either concrete or abstract, either real or imaginary entities and may be either like photographs or motion-pictures or like diagrams, schemas, sketches, symbols. Finally, mental images either may be static or may represent movements and transformations. People, when they think (that is, when they remember, read, listen, speak, make decisions, evaluate, plan, daydream, and so on) may create and manipulate images in their mind. (With "image" we mean all kinds of images described above.) A case where this may occur is problem solving. In fact, when we face a problem, it may be that, while we are thinking, we have in mind, spontaneously or intentionally, one or more images related to that problem. This may facilitate or not the solution of the problem.[3]

[3]This description was aimed at presenting a broad definition of imagery and referred to all kinds of images, so that all possible ways of visualising the problems were included. Furthermore, this definition used "neutral" terms (for instance, "to represent") which were less likely—as compared to other terms (for example, "to reproduce", to "depict", or "to picture")—to orientate participants toward a specific meaning and no examples were given, to circumvent possible biasing effects.

The task was to evaluate how useful, in the participant's opinion, mental imagery was to solve each problem reported in the booklet. In this study, as well as in the subsequent ones, participants were explicitly and repeatedly told that they were not to try to solve problems and that they had to judge imagery usefulness on the basis of what they believed because this was the focus of the study.

For each problem participants had two minutes[4] to read the text (and, in geometrical problems, to look at the pictures) and to rate visualisation usefulness on a 5-point scale (1= minimum: 5= maximum) reported at the bottom of each page. This was considered a "subjective" measure of the imagery effect. At the end of the task participants were asked to write down for which reasons, in general, mental visualisation can help a person to solve problems. The booklet was administered in the university campus during spare time; participants were studied in small groups of four–five individuals each.

Results and Discussion

A three-factor ANOVA model—2 (male vs female) × 3 (Economics vs Humanities vs Social Sciences) × 3 (first vs second–third vs fouth year of course)—was applied in order to assess the influence of individual differences on utility ratings. Twelve separate analyses were carried out, each for each problem. In no case did significant principal effects due to gender, the kind and the year of course attended, and interaction effects emerge.

A one-way repeated-measure ANOVA, assuming the problems as the independent variable, showed that utility ratings differed significantly [$F(11,979) = 15.12$, $MSE = 1.23$, $P < .001$]. More precisely, according to a post hoc contrast analysis, two clusters of problems emerged: problems whose mean subjective ratings were about 3.00 or less (ranks 7–12) and problems with mean ratings above such a cut-off value (Table 1). Visualisation was considered more useful in practical problems and less useful in logical problems; geometrical problems, involving both conceptual and spatial aspects, obtained intermediate ratings. Imagery utility was rated low in problems with only abstract elements (in general, logical, and mathematical problems). Presumably the missionary problem (a logical problem) was rated high because it, but not the other two logical problems, involves a practical aspect (the river crossing) and because it describes a concrete scenario. The high-tide problem (a mathematical problem) was rated high

[4]During pilot administrations of the task, the time available to answer was varied in order to identify which interval was enough to allow students to read the text of the problem and to make their judgements, but not to begin to solve the problem: two minutes seemed to be the interval that best fitted these requirements.

for the same reason: In fact, it describes a detailed scenario, unlike the other two mathematical problems whose instructions only sketched the scenario. Thus, an aspect influencing usefulness ratings seems to be the sketched vs pictorial description of the problem. As far as geometrical problems were concerned, in the two-square task imagery was considered useful because the problem requires no formula, but only to move matches in order to reach the solution. In other words, when a conceptual aspect is involved in the problem, individuals are induced to believe that visual images are less relevant to achieve the solution.

Factor analysis, carried out on the utility ratings in all problems according to the principal component model, led, through the application of the Scree test, to the extraction of three factors (whose eigenvalues were, respectively, 1.88, 1.82, and 1.34). The inspection of the matrix which resulted after Oblimin rotation (Table 1) suggests that Factor 1 was loaded mainly by geometrical, Factor 2 by practical, and Factor 3 by mathematical problems. Logical problems did not constitute an autonomous factor; in fact, the cryptoarithmetic problem was linked to geometrical problems, presumably because of its visuo-spatial layout; the missionary and syllogism problems had similar loadings on two factors.

For each problem an "objective" measure of the strength of the imagery effect was computed by subtracting—when reported—the percentage of solvers in the control condition (where neither imagery instructions nor hints were given) to the percentage of solvers in the

TABLE 1
Imagery Utility Ratings Recorded in Study 1

Problem	*Subjective Rating*		*Factor Analysis Loadings*			*Objective Effect*	*Objective–Subjective Comparison (Rank)*
	Mean (Standard Deviation)	*Rank*	*Factor 1*	*Factor 2*	*Factor 3*		
Two-square	4.17 (1.09)	1	.74	.19	.14	–8%	8 vs 1
Triangle	2.74 (1.09)	8	.71	.20	.27	+ 20%	3 vs 6
Square-rhombus	3.01 (1.32)	7	.55	–.37	–.26	+ 37%	1 vs 5
Cryptoarithmetic	2.70 (1.36)	9	.47	–.14	.04	+ ?	missing data
Syllogism	2.35 (1.31)	11	.31	.05	.26	+ ?	missing data
Pea	4.17 (0.81)	1	.17	.67	.18	+ 10%	5 vs 1
Rope	3.98 (0.99)	3	–.02	.61	–.10	+ 25%	2 vs 3
Candle	3.69 (1.00)	5	–.07	.48	–.01	+ 10%	5 vs 3
Sock	2.67 (1.21)	10	–.15	–.10	.70	–17%	9 vs 7
High-tide	3.54 (1.21)	6	.37	–.29	.57	+ 15%	4 vs 4
Missionary	3.97 (0.97)	4	.02	.53	.57	+ ?	missing data
Alarm-clock	2.14 (1.23)	12	–.03	–.33	.16	+ 9%	7 vs 8

imagery condition (where participants were prompted to use mental visualisation to carry out the task).[5] The difference between the imagery and the control condition was regarded as an "objective" index because it gives a measure of the facilitating effect of visual images as it results from experimental testing.[6]

Because objective measures of imagery efficacy, as well as subjective judgements, vary according to the problem and both can be ranked, it was possible to correlate the two kinds of ratings. Subjective evaluation failed to parallel the objective effects of imagery instructions: The nine problems for which the measure of the actual increase/decrease of solution rates produced by visualisation hints was available were selected and a correlation coefficient between ranks based on such objective utility measure and ranks based on subjective ratings was computed but it was not statistically significant (rho = .06).

In the final interview, students mentioned no specific function played by imagery. They reported, for instance, that visualisation might be useful because it allows one to go "inside" the problem, or to "see" the problem. Briefly, data mirrored what emerged from utility ratings: Mental visualisation was considered beneficial primarily because it allows one to represent the scene of the problem, that is, to give a vivid representation of information which is already pictorial in its own nature. The possibility of mentally recoding elements that are not originally visual into a visual format, to process the data of the problem simultaneously, to transform the situation, and so on, were neglected.

In conclusion, results support the notion that the representation of imagery usefuless in problem solving is affected neither by gender nor by the course of study attended by participants. Undergraduates rated imagery as useful above all when problems deal with concrete situations, whereas mental images were not perceived as possible aids for abstract problems. On these grounds, the subjective evaluation of the imagery utility did not correspond to the objective effect assessed by experiments, which showed that mental visualisation has a beneficial influence also on conceptual tasks.

[5]If a problem was used in two or more experiments, the mean value of the effects recorded in all experiments was computed.

[6]Studies employed to assess the objective imagery usefulness were the following: square-rhombus problem (Antonietti et al., 1994), two-square problem (Antonietti et al., 1994), triangle problem (Barolo, Antonietti, Cecchini, & Stramba-Badiale, 1991), sock problem (Antonietti, 1991), high-tide problem (Antonietti, 1991; Antonietti et al., 1994), alarm-clock problem (Antonietti, 1991), candle problem (Antonietti et al., 1994), rope problem (Antonietti, 1991; Antonietti et al., 1994; Martinsen & Kaufmann, 1991), pea problem (Antonietti, 1991). Studies supporting the facilitating effects of visualisation in logical problems were: syllogistic problem (Frandsen & Holder, 1969), cryptoarithmetic problem (Jaušovec, 1994), missionary problem (Jaušovec, 1994).

STUDY 2

Someone might argue that in Study 1 participants' responses were not based on their opinions about the usefulness of visualisation in problem solving, but on retrospection, that is on what they had perceived about their own mental processes while they were trying, violating instructions, to solve the problems. There are reasons to raise doubts about such a criticism. In fact, during the task the experimenter looked at participants and in no case did she observe behaviours (for example, writing, drawing diagrams, using fingers to count, pointing out the pictures, if provided, or their elements, and so forth) which could suggest that students were engaged in solving the problems. Furthermore, when, at the end of the task, participants were given the solutions to the problems they had read previously, the experimenter observed no reaction (for instance, "Aha" experience expressions) suggesting that students had attempted to solve the problems while they were engaged in rating imagery usefulness. However, the possibility that participants had not followed the prohibition to solve problems can not be completely discarded. In order to reduce such a possibility and to "compel" students to rate mental visualisation usefulness without trying to solve the problems, a different procedure has been devised. Thus, the first goal of Study 2 was to replicate previous findings through a new procedure.

Because in Study 1 responses given to the final question about the reasons of imagery efficacy in problem solving were rather vague, the second purpose was to obtain more precise information about such reasons. The third aim was to assess the relationships between usefulness rating and some individual differences in imagery.

Method

Participants. Fifty undergraduates (twenty-one males and twenty-nine females; aged 19–24) in different disciplines volunteered in the study. As far as the kind and the year of course attended were concerned, they were classified according to the same criteria as in Study 1. Students were neither paid nor received credits for their participation.

Materials and Procedure. Participants were given the same one-page description of the nature and applications of mental visualisation as in Study 1. Eight problems were selected from the 12 used in the previous Study: They were the four problems with the highest and the four with the lowest imagery usefulness ratings. Two problems for each of the four categories resulted in being chosen. Each problem was reported on a card. The task was to arrange two sets of cards, one including problems in which visualisation was considered useful, the other including problems in which visualisation was considered not useful. Ten minutes were allowed for this task. Because of the nature of the task and of the restrictions, participants' attention was focused on classifying problems, with no possibility of spending time by attempting to solve them. A list of possible reasons, derived from the literature about the relationships between imagery and problem solving, was devised; some filler (namely, not supported by the literature) reasons were included in such a list. After the categorisation task, the list—in which the sequence of the nine statements was randomised—was given and participants had to check reasons they endorsed. Finally, participants were administered three imagery questionnaires:

- the Vividness of Visual Imagery Questionnaire (VVIQ) by Marks (1973)
- a revised version of Gordon's (1949) Test of Visual Imagery Control (TVIC)
- the Visual and Verbal Thinking Strategies Questionnaire (Antonietti & Giorgetti, 1993) aimed at measuring the tendency toward visualisation or verbalisation in everyday thought processes.

Participants were studied individually.

Results and Discussion

Hierarchical log-linear models were employed to assess whether the imagery useful/unuseful classification was independent of individual differences. For each problem a separate analysis was carried out by considering the following variables: useful/unuseful classification, gender, kind of course, and year of course attended by participants. As far as interaction effects between the useful/unuseful classification and the other variables were concerned, in no case were significant Y^2 values found.

Percentages of students who included each problem in the "imagery useful" category are reported in Table 2. A saturated 2 × 8 log-linear model was employed to analyse associations between the useful/unuseful classification and the type of problem. A significant interaction between the two variables was found [$Y^2 = 246.32$, $P < .001$]. The same two clusters of problems as in Study 1 can be identified by inspecting the estimates for parameters reported on the second column of Table 2: significant negative z values (corresponding to high imagery efficacy ratings) were obtained for the two-square, rope, pea, and missionary problems; significant positive z values (corresponding to low ratings) were found for the triangle, sock, alarm-clock, and syllogism problems. A

TABLE 2
Imagery Utility Ratings Recorded in Study 2

Problem	*Subjective Rating*			*Study 1–Study 2 Comparison (Ranks)*	*Objective–Subjective Comparison (Ranks)*
	Percentage	*z*	*Rank*		
Geometrical					
Two-square	93	–5.20**	1	1 vs 1	5 vs 1
Triangle	53	3.19*	6	4 vs 6	2 vs 5
Mathematical					
Sock	56	2.57*	5	5 vs 5	6 vs 4
Alarm-clock	23	8.64**	8	7 vs 8	4 vs 6
Practical					
Rope	86	–3.95**	2	2 vs 3	1 vs 2
Pea	84	–3.54*	4	1 vs 4	3 vs 3
Logical					
Syllogism	28	7.87**	7	6 vs 7	missing data
Missionary	86	–3.95**	2	3 vs 2	missing data

*$P < .01$; **$P < .001$.

significant correlation [rho = .85, $P < .01$], between ranks (by considering only problems employed in both studies) of subjective utility ratings in Study 1 and Study 2 (Table 2) emerged.

Correlations between imagery utility judgements in the eight problems were computed. Significant Pearson's coefficients were found in the following pairs of problems: pea/rope [r = .61, $P < .01$], rope/sock [r = .40, $P < .01$], and two-square/triangle [r = .29, $P < .05$]. Also in Study 2 objective and subjective rankings were not correlated (rho = .03).

Log-linear models were used to study associations between the endorsement/non-endorsement of each of the nine statements about why imagery can help problem solving and gender, kind, and year of course attended by participants. No significant Y^2 values were obtained. This leads to the conclusion that opinions about reasons of imagery efficacy are also not affected by individual differences. Responses given to the checklist about the reasons of imagery utility (Table 3) confirmed that visualisation is mostly conceived as a strategy to produce a pictorial representation of the problem. In other words, mental images are considered effective if they help to "see" the problem better. This occurs when visualisation allows one to represent problems in a highly vivid way and/or to realise the structure of the problem. Imagery is not recognised as a transforming, restructuring strategy. Factor analysis was applied to the endorsement (scored as 1)/non-endorsement (scored as 0) of the imagery utility statements. According to the Scree test, three factors were extracted (eigenvalues were, respectively: 1.79, 1.37, and 1.21) and an Oblimin rotation

TABLE 3
Reasons for Imagery Utility Endorsed in Study 2 and Study 3

"Mental Visualisation is useful in problem solving because it allows one to . . ."	*Study 2*				*Study 3*
	Percentage of Participants Who Endorsed the Statement	*Factor*	*Analysis*	*Loadings*	*Total Number of Times of Statement Endorsement*
Overcome fixedness and mechanicity in thinking	16	.79	.11	–.03	24
Transform the problem easily and in unusual ways	20	.78	–.02	.05	68
Memorise the elements of the problem easily	48	–.65	.09	.06	not presented
Have in mind different elements of the problem simultaneously	64	–.16	.75	–.32	75
Have a vivid and detailed representation of the problem	64	–.09	.65	.12	68
Have a sense of certainty	18	–.20	–.51	–.07	not presented
Stress the essential aspects of the problem	56	–.08	.28	.76	32
Have a concrete representation of the problem	74	–.15	–.16	.59	not presented
Highlight relations between the elements of the problem	64	.18	–.01	.40	92

produced the matrix reported in Table 3. It is worth noticing that the two filler statements ("Visualisation allows one to memorise the elements of the problem easily" and "Visualisation gives a sense of certainty") had negative loadings on all factors. Factor 1 could be labelled as Flexibility Effect, Factor 2 as Seeing Effect, and Factor 3 as Analysing Effect. In each problem, chi-square tests revealed no significant associations between the useful/unuseful classification and the endorsement/non-endorsement of particular reasons.

Finally, individual differences in imagery were related neither to the subjective evaluation of imagery usefulness nor to the opinions about reasons for imagery utility. In each imagery questionnaire participants were classified as low, medium, or high visualisers according to the total scores obtained in that questionnaire and by assuming, respectively, the 33rd and the 66th percentiles of the distribution of the total scores as cut-off points; in each problem cross-tabulation between this classification and the imagery useful/unuseful categorisation and the endorsement/non-endorsement of each statement failed to produce statistically significant chi-square values.

So, even though the evaluation procedure was changed, people revealed a metacognitive representation of imagery usefulness that is consistent with the picture emerging from Study 1. Such a representation seems to be internally coherent and is not affected by individual differences. Responses given both in the classification task and in the statement-checking task revealed that students think that mental visualisation is above all a strategy aimed at providing a clear picture of the problem; this is particularly beneficial when problems are already pictorial in their own nature. Because other functional properties of visual images are neglected in people's representation, a discrepancy between the subjectively and objectively estimated measures of efficacy emerges.

STUDY 3

In Studies 1 and 2 subjects were requested to evaluate the utility of mental visualisation in a problem on the basis of the idea that imagery, in some circumstances, may help the solution. However, given a problem, different images can be constructed; further, they can be transformed mentally in different ways. Students might fail to identify the heuristic functions of visualisation in general, but they might succeed if the description of a specific imagery strategy is provided. Study 3 was carried out to verify this conjecture. A second goal was to assess whether undergraduates are able to recognise why an imagery operation facilitates reasoning if they are asked to do so in reference to a specific problem but not, as in Study 2, to problem solving in general.

Method

Participants. Fifty undergraduates (twenty-three males and twenty-seven females, aged 19–23 years) in different disciplines volunteered in the study. They were classified according to the course of study (Natural vs Social Sciences) and to the year (first–second vs third–fourth) attended. They were neither paid nor received credits.

Materials and Procedure. Participants were given the same one-page description of the nature and applications of mental visualisation as in Study 1. The same problems as in Study 2 were presented in a booklet. The text of each problem was followed by instructions to visualise. For instance, in the sock problem instructions were: "Visualise mentally each sock you take out from the drawer". Participants were told that such instructions had been devised in order to carry out an experiment to verify whether hints at visualising increased the amount of solutions as compared to a control condition where such hints were not given. Participants were requested to anticipate the results of such an (imaginary) experiment, that is, to check problems whose solution, in their opinion, should be facilitated by visualisation instructions. Two minutes were allowed for each problem. Then participants were invited to consider again the problems they had checked; a list of six reasons for imagery efficacy—selected from among the nine reasons employed in Study 2 (see Table 3)—was presented and for each of the checked problems participants had to choose which reasons they thought to be responsible for the facilitating effects of visualisation in that problem. Ten minutes were allowed for this task. Participants were studied individually.

Results and Discussion

Also in this study, the application of a hierarchical log-linear model supported the independence of the imagery usefulness categorisation of individual differences. Separate analyses were carried out for each problem and in no case did significant interaction effects between the useful/unuseful classification and gender, kind, and year of course attended by participants emerge.

By employing a saturated 2 × 8 log-linear model, a significant interaction [$Y^2 = 84.00$; $P < .001$], between the useful/unuseful classification and the type of problem emerged. Three clusters of problems can be detected (Table 4): problems (missionary and pea) with significant negative *z* values; problems (rope, tow-square, and syllogism) with no significant *z* values; and problems (triangle, sock, and alarm-clock) with significant positive *z* values. "Pictorial" problems were again the most frequently recognised problems in which visualisation is beneficial. However, differences among problems were smaller than in Studies 1 and 2. This might depend on the fact that the description of specific imagery strategies—which otherwise participants should have never thought of—led them to overestimate the role of visualisation in the problem solution process; in fact, rates of imagery utility endorsement were never lower than 45%. In other cases such a description might have cast doubts about the imagery efficacy; for instance, in the two-sqaure problem instructions for

TABLE 4
Imagery Utility Ratings Recorded in Study 3

Problem	*Subjective Rating*			*Study 1–Study 3 Comparison (Ranks)*	*Study 2–Study 3 Comparison (Ranks)*	*Objective–Subjective Comparison (Ranks)*
	Percentage	*z*	*Rank*			
Geometrical						
Two-square	64	0.30	4	1 vs 4	1 vs 4	5 vs 3
Triangle	55	2.25*	6	4 vs 6	6 vs 6	2 vs 4
Mathematical						
Sock	45	4.33**	7	5 vs 7	5 vs 7	6 vs 5
Alarm-clock	45	4.33**	7	7 vs 7	8 vs 7	4 vs 5
Practical						
Rope	67	–0.36	3	2 vs 3	3 vs 3	1 vs 2
Pea	84	–4.08**	2	1 vs 2	4 vs 2	3 vs 1
Logical						
Syllogism	59	1.39	5	6 vs 5	7 vs 5	missing data
Missionary	88	–5.10**	1	3 vs 1	2 vs 1	missing data

*$P < .05$; **$P < .001$.

visualising (“Move matches freely in your mind”) were rather unconvincing because only two matches had to be moved. However, it is also worth noticing in this study that two opposite clusters of problems (having, respectively, significant negative and positive estimates for parameters) emerged and that problems included in these clusters were included in the corresponding clusters in Studies 1 and 2. Furthermore, ranks of subjective usefulness in Study 3 were significantly correlated with their counterparts both in Study 1 [rho = .72, $P < .01$], and in Study 2 [rho = .71, $P < .01$] (Table 4). As in previous studies, ranks of subjective evaluations of imagery utility were not correlated to ranks of objective evaluations (rho = .55), even though in this case the discrepancy between the two kinds of measure was reduced as compared to Studies 1 and 2.

As far as reasons for efficacy identified for each of the problems were concerned, no statistically significant association (as revealed by analyses based on log-linear models) between their endorsement/non-endorsement and gender, kind, and year of study attended was found. The total number of times that each reason was checked for the eight problems was computed (Table 4, last column). This measure yielded a pattern which is not substantially different from Study 2. Reasons were ranked according to the frequency with which they were checked in both Studies 2 and 3: A significant correlation [rho = .80, $P < .05$], was found. Cross-tabulation of imagery useful/unuseful responses given to each couple of problems and cross-tabulation of endorsement/non-endorsement of each statement for that couple of problems were carried out to identify possible links: Chi-square tests failed to support any association.

DISCUSSION

According to Katz (1983), individual differences in the use of mental imagery in cognition depend, besides visuo-spatial skills and on encoding preferences, on metacognition. Thus, we can assume that the effective use of mental images in problem solving is modulated by imagery abilities, by habits and preferences for following visual strategies in thinking, and by naive conceptions about imagery and its functions. As far as the last aspect is concerned, we can hypothesise that sensitivity about knowing when imagery might be useful leads a person to succeed in representing and processing visually the problems which he or she faces. The lack of such a sensitivity decreases the potential effects of imagery. In fact, studies on the spontaneous use of imagery in problem solving showed that, if instructions about when and how to visualise are not given, solution rates are not always enhanced (Antonietti, Scafidi, & Resinelli, 1997). This stresses the need to assess whether people have relevant metacognitive beliefs about visualisation. The paper addressed such a question. More precisely, we can assume that persons have a metacognitive representation of imagery usefulness, but no previous study told us *what* such a representation is like; the investigation was aimed at highlighting the features of this representation.

Results of the three studies carried out suggest that gender, kind, and year of course attended by students do not affect the representation of the role of mental visualisation in problem solving. Furthermore, this representation is not related to imagery vividness, imagery control, and visualiser-verbaliser cognitive style. Presumably, individual differences in imagery experience and in imagery habits play a minor role in determining beliefs about the efficacy of mental visualisation. Such beliefs seem to be based on generalised assumptions concerning the features of the external stimuli rather than personal (cultural and psychological) features.

Data acquired through different methodologies were consistent even though procedures were changed: In all studies similar clusters of problems emerged and for each problem approximately the same ranks were recorded. Factor analyses showed that students' opinions reveal a structure that can be interpreted in terms of coherent sub-components, corresponding to specific sets of problems. This leads us to maintain that the subjective evaluation of the imagery usefulness is grounded on a stable and shared system of metacognitive beliefs. This finding seems to be interesting because there are fields in metacognition research where a clear structure underlying people's beliefs fails to emerge.

The subjective evaluation does not match the objective influence of visualisation on problem solving. Experimental findings highlighted that the detailed and particular nature of some realistic images is detrimental in problem solving because they introduce distracting elements. On the other hand, the literature suggests quite clearly that imagery does not impede abstract thinking and that the functional usefulness of visual images is not tied to the handling of concrete, perceptual material: on the contrary, visualisation is also highly beneficial under abstract task conditions. However, individuals consider mental visualisation as useful above all when the problem deals with "pictorial" information. As far as conceptual problems are concerned, mental images are not perceived as a possible aid. This is consistent with the finding that people argue that imagery is useful when it represents concrete rather than abstract or verbal material (Antonietti, Giorgetti, Resinelli, & Scafidi, 1995). So, imagery seems to be conceptualised as a strategy to process data that are visual or spatial in nature, but not to process non-visual elements that might be coded or recoded in a visual format. Furthermore, visualisation is considered productive above all in problems with few conceptual components; when conceptual features are blatant (as in the logical and mathematical problems or in the geometrical problems involving the application of formulae), individuals are reluctant to assign an effective role to imagery. All this can give an account of the mismatch between the experimentally tested and the subjectively estimated effects of visualisation.

These beliefs are mirrored by opinions about the reasons that, according to students, support imagery use in problem solving. People consider visualisation above all as a mental operation that allows one "to see" the problem or "to see" within it; they have in mind only a static function of images. The possibility to transform the problem with the mind's eye—for instance, by moving, turning, or combining its elements—is unrecognised. It is difficult to maintain that this metacognitive representation was biased by the instruction given, because, in the description of imagery and of its occurrence in thinking, no elements should orientate participants toward a narrow interpretation of mental visualisation. In fact, in such a description not only the "pictorial", static, realistic aspects of images, but also the possibility of generating schematic, dynamic, and "conceptual" images and the possibility of representing abstract entities in images were mentioned.

Which consequences may derive from people's beliefs about imagery usefulness? Students claim that visual images are beneficial primarily in order to have in mind a clear and vivid picture of the scene described in the text of the problem. This restricts the range of situations in which visualisation may occur. Thus, only a set of problems—namely, those which are originally "pictorial" in their own nature—are putative candidates to benefit from imagery and, consequently, only in these circumstances are people likely to employ visual images in reasoning. Moreover, such kinds of "mental pictures" are not always—as empirical reports showed—useful in problem solving, so that subjects do not experience positive effects of visualisation and, consequently, are discouraged from using it again in the future. By contrast, the possibility to represent through visual images contents that are not originally visual—such as logical or mathematical relationships—tend to be neglected by people; therefore, situations in which—as experiments demonstrated—visualisation could be useful are not the most frequently experienced.

In conclusion, the data has stressed the inadequacy of people's conception about the role of imagery in problem solving because they are led to ignore a potential cognitive resource. This raises the need to train individuals to benefit from mental visualisation by modifying some generalised beliefs concerning imagery which appear to be far from what experiments show about the possibility of discovering problem solutions through the mind's eye. Metacognitive knowledge about imagery can be changed (Denis & Carfantan, 1990). A promising direction seems to be that of presenting expert models of visualising. In fact, as shown by Study 3, if a specific imagery strategy is described, the discrepancy between subjective and objective measures of imagery efficacy decreases.

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